From Eqs. (2) and (3) we obtain

$$\lambda = \left(0{,}40\frac{T}{T_1} + 0{,}59\right)(An + B)\cdot 10^{-3}. \qquad (4)$$

With the aid of Eq. (4) one can calculate the effective thermal conductivity coefficient of aluminum oxide with various copper contents as a function of temperature without experimental study. For such a calculation only a knowledge of the copper content is required.

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IDENTIFICATION OF A SPECIFIED THERMAL REGIME IN A STRUCTURE ON THE BASIS OF EXPERIMENTAL DATA OBTAINED IN OTHER REGIMES

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A method is proposed for calculating the temperature at a given point of a complex structure with a specified heating regime on the basis of experimental data obtained in other regimes.

During the development and testing of new equipment and the modernization of existing equipment operating at elevated temperatures, it is necessary to determine the temperature at individual points inside the structure in given thermal regimes. Difficulties in allowing for all features of heat transfer make it difficult to solve this problem by direct numerical methods through solution of the heat-conduction equation for complicated, multilayered structures. The quality of the results obtained is significantly affected by the shortage — and in some cases, complete lack — of information on the laws of distribution of contact heat-transfer resistance between the layers and heat transfer in air gaps.

In connection with this, it is very important to determine the temperature inside an object on the basis of temperature data obtained during experiments in other heating regimes. Searches for a solution to this problem have led to the idea of replacing the actual complex structure by a simpler mathematical model with fewer layers characterized by a certain effective heat-transfer coefficient [1, 2].

The method employed in [1] is based on the use of a so-called "reference" regime and conversion factor in the calculation of prescribed surface temperature regimes. The conversion factor is calculated from known empirical temperature data at a given point of the structure by analytical solution of a unidimensional heat-conduction equation for a one-layer wall. This method gives good results in several cases. However, it has certain limitations in terms of its application, indlucing the fact that it is possible to calculate only monotonic regimes the length τ^e of which does not exceed the length of the reference regime.

Another study [2] proposed that the computational model be the heat-conduction equation for fewer number of layers of the same geometry, with a certain constant effective thermal

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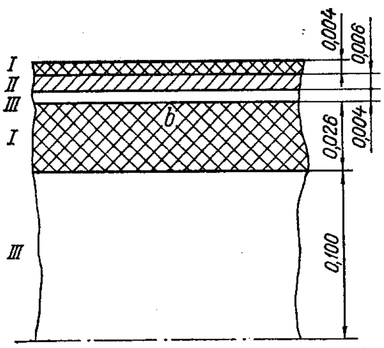

Fig. 1

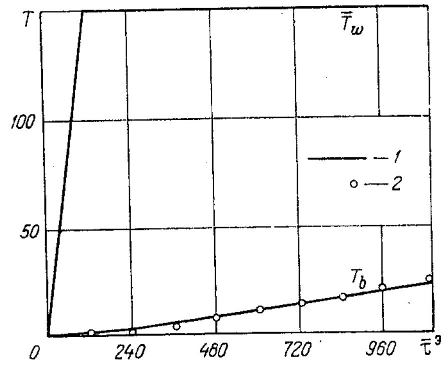

Fig. 2

Fig. 1. Diagram of multilayered cylindrical wall used for the numerical experiment: I) thermal insulation; II) metal; III) air.

Fig. 2. Results of calculation to determine the dependence of temperature T_b, °C, on time τ^e, sec, in a quasisteady heating regime T_w, °C: 1) experimental data; 2) theoretical relation.

TABLE 1. Values of Effective Thermal Conductivity λ^e, W/m·deg, for a Point of a Structure with the Coordinate b = 0.026 m

T_w, °C	τ^e, sec		
	720	1200	1800
100	5,134	4,629	4,389
200	5,379	4,986	4,855
300	5,855	5,618	5,651
400	6,403	6,346	6,575

conductivity λ^e for each layer. It was concluded from the study results that when it is necessary to determine the temperature at one point rather than the entire temperature profile for the cross section, it is possible to replace the actual structure by a one-layer computational model with a coefficient λ^e = const.

This conclusion is in full accord with data we obtained during analysis of the results of calculation of heat transfer in a cylindrical multilayered wall with a mathematical model in the form of a unidimensional heat-conduction equation for a one-layer plane wall. Thus, in several cases where the method in [1] cannot be used, it is possible to solve the formulated problem by representing heat transfer in the structure by means of an abstract mathematical model described by the heat-conduction equation:

$$c\rho \frac{\partial T(x, \tau)}{\partial \tau} = \lambda^e \frac{\partial^2 T(x, \tau)}{\partial x^2}, \tag{1}$$

$$0 < \tau \leqslant \tau^e, \; 0 < x < d, \qquad T(x, 0) = f(x), \; 0 \leqslant x \leqslant d, \tag{2}$$

$$T(0, \tau) = T_w(\tau), \; q(d, \tau) = 0, \; 0 \leqslant \tau \leqslant \tau^e, \tag{3}$$

where $c\rho$ = const, λ^e = const.

Mathematical model (1)-(3) is used to identify the thermal regime of an actual structure at a specified point x = b, $0 < b \leq d$ by calculating the effective thermal conductivity λ^e = const. This effective thermal conductivity ensures the prescribed empirical relation $T^e(b, \tau)$ at this point. The coefficient λ^e is determined by solving the inverse coefficient problem of heat conduction using an iterative method for minimization of a functional

$$J_k = \frac{1}{\tau^e} \int_0^{\tau^e} [T_k(b, \tau) - T^e(b, \tau)]^2 \, d\tau, \tag{4}$$

where k is the number of the iteration.

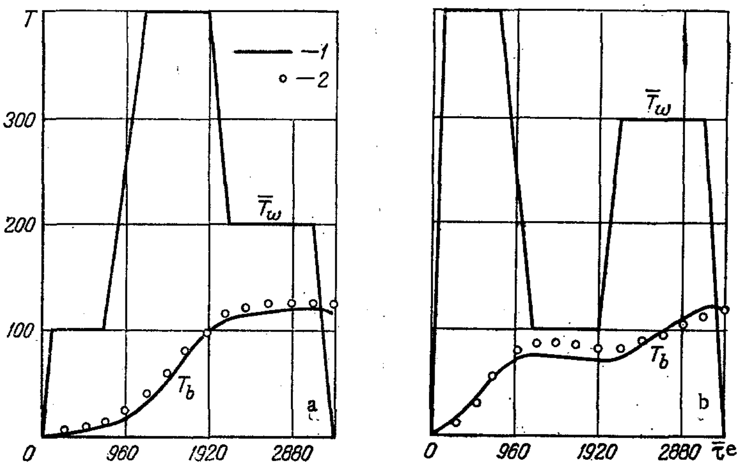

Fig. 3. Result of calculation of the dependence of temperature T_b on time $\bar{\tau}^e$ with a nonsteady heating regime I (a) and II (b), $\bar{T}_w$, °C: 1) experimental data; 2) theoretical relation.

TABLE 2. Values of Effective Thermal Conductivity $\bar{\lambda}^e_i$ for Calculating Nonsteady Regimes I and II. $\bar{\lambda}^e_i$, W/m•deg; $\bar{\tau}^e_i$, sec

Regime	$\bar{\tau}^e_0$	$\bar{\lambda}^e_1$	$\bar{\tau}^e_1$	$\bar{\lambda}^e_2$	$\bar{\tau}^e_2$	$\bar{\lambda}^e_3$	$\bar{\tau}^e$
I	0	5,134	720	6,438	2160	4,989	3360
II	0	6,346	1200	5,134	1920	5,631	3360

The effective thermal conductivity λ^e is calculated from the formula

$$\lambda^e_k = \lambda^e_{k-1} + \alpha_k p_k, \tag{5}$$

where p_k is the direction of the slope calculated by the method of conjugate gradients [3] and α_k is the step of the slope, calculated from the condition

$$J_k = \min_{\alpha} J(\lambda^e_{k-1} + \alpha p_k). \tag{6}$$

The gradient of the function J'_k needed to calculate the direction of the slope is determined from the solution of the corresponding conjugate problem. Different methods of finding J'_k and α_k are discussed in [4, 5].

The iteration performed to solve the inverse heat-conduction problem is stopped in accordance with the condition

$$\sqrt{J_k} - \sqrt{J_{k-1}} \leqslant \varepsilon,$$

where ε is a small number dependent on the desired accuracy of the calculated result (one usually takes $\varepsilon = 0.001$°C).

As a measure of adequacy of the calculated thermal conductivity λ^e from the mathematical model and actual heat transfer at the specified point of the structure we take the absolute maximum deviation of the theoretical temperature from the experimental temperature:

$$S = \max_{\tau \in [0, \tau^e]} |T(b, \tau) - T^e(b, \tau)|.$$

It is understood that it is desirable to obtain a solution to the inverse problem that will be characterized by as small a value of S as possible.

During the development of the above method, we studied the possibility of recalculating empirical data for quasisteady and nonsteady regimes of different duration. Here, we used mathematical model (1)-(3) to identify heat-transfer processes both in actual structures and in cylindrical multilayered mathematical models.

Analysis of the results of the identification obtained from laboratory and numerical experiments for quasisteady heating regimes showed the following.

1. The value of thermal conductivity λ^e depends not only on the location of the test point of the structure $x = b$ and the limiting temperature of the surface, but on the duration of the time interval for which the calculation was performed (Table 1).

2. The value of thermal conductivity λ^e is also affected by the temperature dependence on the initial section of the regime. Thus, a more adequate mathematical model could be obtained by increasing the accuracy of measurement on the initial section. At the same time, the calculations showed that it is possible to refine the model by means of a corresponding change in the temperature relation $T^e(b, \tau)$ on this section and a corresponding reduction in S.

3. Model (1)-(3) makes it possible to sufficiently accurately identify unidimensional heat transfer in a multilayered structure. In the case of two- and three-dimensional heat transfer, a mathematical model with an effective thermal conductivity λ^e = const cannot be used because the relation $T(b, \tau)$ calculated from it does not adequately reflect the relation $T^e(b, \tau)$ measured in the experiment.

In the case of the use of model (1)-(3), the solution of the problem can be broken down into three stages: the first stage involves calculation of λ^e = const for the specified point of the structure from data from several experiments in quasisteady regimes $T^e(b, \tau)$ with different durations. This calculation is performed by the method of solving inverse coefficient problems of heat conduction. Then tables of values of $\lambda^e(b, T_w, \tau^e)$ are compiled. The second stage involves the use of the table of $\lambda^e(b, T_w, \tau^e)$ and interpolation or extrapolation to find $\bar{\lambda}^e$ for the specified quasisteady regime $\bar{T}_w$ of duration $\bar{\tau}^e$; the third stage entails calculation of the temperatures at the specified point $x = b$ in the thermal regime $\bar{T}_w(\tau)$ over the time $\bar{\tau}^e$ from model (1)-(3) with the use of the value of $\bar{\lambda}^e$ found from the table.

It is necessary in the calculations to consider the results obtained with extrapolated values of $\bar{\lambda}^e$ are usually less accurate than results obtained with interpolated values. Nonetheless, despite their approximate character, they make it possible to evaluate the thermal state of the structure in the new regime.

To obtain more accurate values of temperature in the structure for the new quasisteady regime, it is desirable to make the initial section of the relation $\bar{T}_w(\tau)$ equal or nearly equal to the initial section of the experimental regimes.

Calculations performed on the basis of data from numerical experiments showed that it is possible to use the table of values of $\lambda^e(b, T_w, \tau^e)$ obtained for several quasisteady regimes to calculate the temperature $T(b, \tau)$ at a given point of a structure with a nonsteady heating regime $\bar{T}_w(\tau)$.

Experiments conducted with mathematical model (1)-(3) showed that the coefficient $\bar{\lambda}^e$ in this case should be given in the form of a step function $\bar{\lambda}^e$ $(\bar{\lambda}^e{}_1, \ldots, \bar{\lambda}^e{}_i, \ldots, \bar{\lambda}^e{}_I)$ which changes in accordance with the temperature of the heating surface. To choose values of $\bar{T}_{wi}$ and $\bar{\tau}^e{}_i$ for determining $\bar{\lambda}^e{}_i$, it is necessary to study the prescribed nonsteady temperature regime on the surface $\bar{T}_w(\tau)$ and to isolate several time intervals during which the regime $\bar{T}_{wi}(\tau)$ can be taken as quasisteady. Having used the superposition principle, we can assume that heating during each previous interval creates the initial temperature field for the next interval, and we can examine heating on each interval independently of the adjacent intervals. Given this formulation, we determine $\bar{\lambda}^e{}_i$ on each interval from the table of values $\lambda^e(b, T_w, \tau^e)$ with the use of the corresponding values of temperature $\bar{T}_{wi}$ and time $\bar{\tau}^e{}_i = \bar{\tau}_i - \bar{\tau}_{i-1}$, where $i \in [0, I]$ is the number of the interval; I is the number of intervals.

As an example, we will present results of calculations of the temperature at a specified point in the case of one quasisteady and two nonsteady regimes. The temperature is calculated from data from numerical experiments in which the model was multilayered cylindrical wall (Fig. 1). We studied the thermal state of the structure at the point $b = 0.026$ m. The determination of the thermal state at this point by direct computational methods was complicated by the presence of an air interlayer in which the features of heat transfer were unknown. A table of values of $\lambda^e(b, T_w, \tau^e)$ was obtained from the numerical experiments for quasisteady regimes $T_w(\tau)$ = 100, 200, 300, and 400°C with a maximum duration τ^e = 1800 sec. Here we solved a series of inverse heat-conduction problems for specified temperature regimes and the times τ^e = 720, 1200, and 1800 sec (Table 1).

Figure 2 shows the calculated result for the quasisteady regime $\bar{T}_w = 150°C$ with $\bar{\tau}^e$ = 1080 sec, $\bar{\lambda}^e = 4.921$ W/m•deg, and $c\rho = 10^8$ J/kg•deg. The value of thermal conductivity $\bar{\lambda}^e$ was determined by interpolation from Table 1.

Values of effective thermal conductivity in Table 2 were used to calculate the temperature relation $T(\tau)$ at the point b = 0.026 m for nonsteady regimes (Fig. 3). In determining $\bar{\lambda}^e{}_i$, we broke each of the nonsteady regimes $\bar{T}_w(\tau)$ down into three sections with respect to time. In each section, heating was assumed to be quasisteady. Accordingly, each section in Table 1 was calculated by interpolation of the thermal conductivities.

Comparison of the results calculated by simplified model (1)-(3) and data from a numerical experiment on a multilayered cylindrical shell showed that the error of the relation found here $T(\tau)$ is within the permissible range.

NOTATION

τ^e, Duration of regime; λ^e, effective thermal conductivity; $T(x, t)$, temperature; x, running coordinate; τ, running time; c, specific heat; ρ, density; d, thickness of plate; q, heat flux; T_w, temperature of heated wall.

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GRID METHOD FOR CALCULATION OF FLOW AND HEAT EXCHANGE OF A VISCOUS INCOMPRESSIBLE LIQUID

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An explicit difference method is described for calculation of the flow and heat exchange of an incompressible liquid, allowing calculations at quite large Reynolds numbers.

The development of simple and effective numerical methods for modeling liquid flow and heat exchange processes at high Reynolds and Grashof numbers is of great importance in many fields of contemporary technology. Use of the algorithm presented in [1] by the present author, involving a scaled explicit difference scheme, permits successful solution of the boundary-layer problem and natural convection of a gas [2].

The present study will offer a numerical method based on the scaled difference scheme for calculating flow and heat exchange of a viscous incompressible liquid over a wide range of Grashof and Reynolds numbers.

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